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# The Pattern of Neutrino Masses and How to Determine It\*

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## I INTRODUCTION

Our knowledge of the neutrino sector of the Standard Model has recently undergone a revolution. Deficits of the atmospheric muon neutrino flux and the solar electron neutrino flux compared to their predicted values can be understood in terms of neutrino oscillations [1] and we can therefore infer that neutrinos have non-degenerate masses. Additional but somewhat less secure evidence for  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  and  $\nu_\mu \rightarrow \nu_e$  oscillations has been found in the LSND accelerator experiment [1]. Because these experiments have widely different  $L/E_\nu$  ranges ( $\approx 10$  to  $10^4$  km/GeV for atmospheric,  $\approx 10^{11}$  for solar, and  $\approx 1$  for LSND), the mass-squared differences required to explain the phenomena must be distinct. Given the observations, an important next step is to deduce the pattern of neutrino masses and mixings. Such studies depend on the number of neutrinos. The invisible width of the  $Z$ -boson measured in LEP experiments gives  $N_\nu = 2.993 \pm 0.011$ , consistent with the usual  $\nu_e, \nu_\mu$  and  $\nu_\tau$  “active” neutrinos. But there may also be right-handed “sterile” neutrinos with no weak interactions. Only the observation of oscillations of the active neutrinos to sterile neutrinos can test for their existence. In the following, we first discuss the atmospheric and solar neutrino data in a 3-neutrino framework and then later generalize our considerations to include the LSND data with oscillations of four neutrinos. The proceedings of the Neutrino 98 conference [1], a current review by the organizer of COSMO 98 [2], and recent phenomenological analyses [3–9] may be consulted for references to the vast primary literature.

When neutrino flavor eigenstates  $\nu_f$  are not the same as the mass eigenstates  $\nu_i$ , e.g., for two neutrinos,

$$\nu_f = \cos \theta \nu_1 + \sin \theta \nu_2, \quad \nu_{f'} = -\sin \theta \nu_1 + \cos \theta \nu_2, \quad (1)$$

then neutrinos oscillate. The vacuum oscillation probabilities are

$$P(\nu_f \rightarrow \nu_{f'}, L) = A \sin^2 \left( \frac{\delta m^2 L}{4E} \right) \quad \text{“appearance”,} \quad (2)$$

$$P(\nu_f \rightarrow \nu_f, L) = 1 - A \sin^2 \left( \frac{\delta m^2 L}{4E} \right) \quad \text{“survival”,} \quad (3)$$

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where  $A = \sin^2 2\theta$ ,  $\delta m^2 = m_2^2 - m_1^2$ ;  $L$  is the path length and  $E$  is the neutrino energy. The neutrino anomalies can be explained by effective two-neutrino oscillations; vacuum oscillations can account for the solar [10], atmospheric [11] and LSND anomalies; matter-enhanced oscillations [12] are an alternative to explain the solar neutrino deficit.

## II NEUTRINO ANOMALIES AND THEIR OSCILLATION INTERPRETATIONS

Atmospheric Cosmic ray interactions with the atmosphere produce  $\pi$ -mesons and the decays  $\pi \rightarrow \mu\nu$  and  $\mu \rightarrow \nu_e e \bar{\nu}_\mu$  give  $\nu_\mu$  and  $\nu_e$  fluxes in the approximate ratio  $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e) \sim 2$  for  $E_\nu \sim 1$  GeV. Measurements of  $R = (N_\mu/N_e)_{\text{data}}/(N_\mu/N_e)_{\text{MC}}$  for  $E_\nu \sim 1$  GeV find values of  $R \sim 0.6$  [1]. In the water Cherenkov experiments the single rings from muons are fairly clean and sharp, while those from electrons are fuzzy due to electromagnetic showers. The separate distributions of  $\mu$ -like and  $e$ -like events versus the zenith angle establish that the anomalous  $R$ -ratio is due to a deficit of upward  $\mu$ -like events. As suggested long ago [11], the data are well described by  $\nu_\mu \rightarrow \nu_\tau$  or  $\nu_\mu \rightarrow \nu_s$  oscillations with  $\delta m_{\text{ATM}}^2 \approx 3 \times 10^{-3}$  eV $^2$  and  $A_{\text{ATM}} \approx 1$ . For sub-GeV neutrino energies,  $L/E$  is large at  $\cos \theta < 0$  and the oscillations average,  $P(\nu_\mu \rightarrow \nu_\mu) \approx 0.5$ . At multi-GeV energies,  $L/E$  is large at  $\cos \theta = -1$  and  $P(\nu_\mu \rightarrow \nu_\mu) \approx 0.5$ ; also  $L/E$  is small at  $\cos \theta = +1$  and  $P(\nu_\mu \rightarrow \nu_\mu) \approx 1$ .

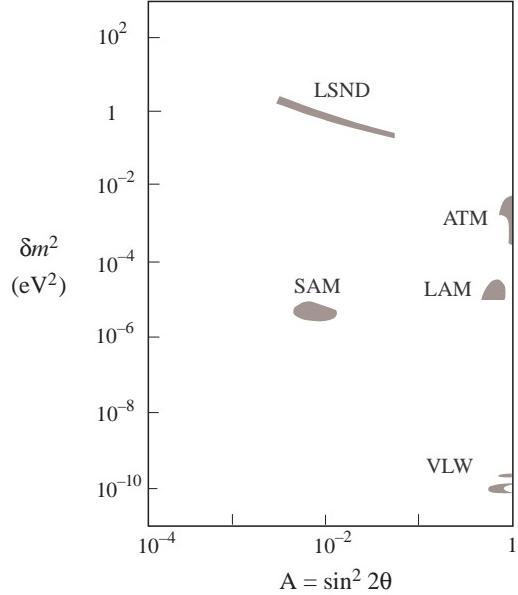
Solar Three types of solar  $\nu_e$  experiments, (i)  $\nu_e$  capture in Cl [Homestake], (ii)  $\nu_e e \rightarrow \nu_e e$  [Kamiokande and SuperKamiokande], (iii)  $\nu_e$  capture in Ga, measure rates below standard model expectations. The different experiments are sensitive to different ranges of solar  $E_\nu$ . Three regions of oscillation parameter space are found to accommodate all these observations [3,7,8]:

	$\delta m_{\text{SOL}}^2$ (eV) $^2$	$A_{\text{SOL}}$
Small Angle Matter (SAM)	$\sim 10^{-5}$	$\sim 10^{-2}$
Large Angle Matter (LAM)	$\sim 10^{-5}$	$\sim 0.6$
Vacuum Long Wavelength (VLM)	$\sim 10^{-10}$	$\sim 1$

LSND The Los Alamos experiment studied  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  oscillations from  $\bar{\nu}_\mu$  of  $\mu^+$  decay at rest and  $\nu_\mu \rightarrow \nu_e$  from  $\nu_\mu$  of  $\pi^+$  decay in flight. The results, including restrictions from BNL, KARMEN and Bugey experiments, suggest  $\nu_\mu \rightarrow \nu_e$  oscillations with

$$0.3 \text{ eV}^2 < \delta m_{\text{LSND}}^2 < 2.0 \text{ eV}^2, \quad A_{\text{LSND}} \approx 4 \times 10^{-2} \text{ to } 3 \times 10^{-3}. \quad (4)$$

Figure 1 illustrates the parameter regions for the solar, atmospheric and LSND oscillation interpretations. Since three distinct  $\delta m^2$  are needed to explain the atmospheric, solar and LSND data, but there are only two independent  $\delta m^2$  from



**FIGURE 1.** Regions of oscillation parameters  $\delta m^2, \sin^2 2\theta$  that can explain the atmospheric, solar, and LSND anomalies.

$\nu_e, \nu_\mu, \nu_\tau$  neutrinos, there are two possible roads to follow: (i) put the LSND anomaly aside until it is confirmed by the KARMEN and mini-BooNE experiments, or (ii) explain all three anomalies by invoking oscillations to a sterile neutrino as well. We consider both routes in the following.

### III NEW INFERRED LIMITS ON NEUTRINO MASS

The fact that the neutron undergoes  $\beta$ -decay implies that the electron-neutrino is a linear combination of one or more mass eigenstates with mass below the kinematic  $\beta$ -decay end-point limit of  $m_\beta = 4.4$  eV. The interpretation of the atmospheric and solar neutrino anomalies in terms of oscillations tells us that the neutrino mass splittings in a three-neutrino universe are much smaller than  $m_\beta$ . We conclude therefore that all three neutrino mass eigenvalues satisfy  $m_j \leq m_\beta$  and that the linear combinations of these mass states which are  $\nu_\mu$  and  $\nu_\tau$  have effective masses  $\leq m_\beta$  as well [13]. These bounds represent a factor of  $10^5$  to  $10^6$  improvement over the current bounds  $m_{\nu_\mu} < 170$  keV and  $m_{\nu_\tau} < 18.2$  MeV. The largest mass eigenvalue is bounded below by  $m_\beta \geq \sqrt{\delta m_{\text{atm}}^2} \geq 0.002$  eV. Generalizing to include  $\nu_\mu \rightarrow \nu_e$  oscillations in the LSND experiment with one sterile and three active neutrinos, we obtain an upper bound of 5.4 eV on all four neutrino masses and a lower bound on the largest mass eigenstate of  $m_4 \geq \sqrt{\delta m_{\text{LSND}}^2} \gtrsim 0.5$  eV.

## IV FROM EFFECTIVE 2-GENERATION TO 3-GENERATION OSCILLATIONS

The neutrino MNS mixing matrix [14], for either Dirac or Majorana neutrinos, can be parametrized by three angles  $\theta_i$  and a CP-violating phase as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} c_1 c_3 & c_1 s_3 & s_1 e^{-i\delta} \\ -c_2 s_3 - s_1 s_2 c_3 e^{i\delta} & c_2 c_3 - s_1 s_2 s_3 e^{i\delta} & c_1 s_2 \\ s_2 s_3 - s_1 c_2 c_3 e^{i\delta} & -s_2 c_3 - s_1 c_2 s_3 e^{i\delta} & c_1 c_2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (5)$$

where  $c_j \equiv \cos \theta_j$ ,  $s_j \equiv \sin \theta_j$ , and  $\nu_1, \nu_2, \nu_3$  are the mass eigenstates. The vacuum oscillation probabilities of interest are

Atmospheric:

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - (c_1^4 \sin^2 2\theta_2 + s_2^2 \sin^2 2\theta_1) \sin^2 \Delta_{atm} \quad (6)$$

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta_1 \sin^2 \Delta_{atm} \quad (7)$$

$$P(\nu_e \leftrightarrow \nu_\mu) = s_2^2 \sin^2 2\theta_1 \sin^2 \Delta_{atm} \quad (8)$$

$$P(\nu_e \leftrightarrow \nu_\tau) = c_2^2 \sin^2 2\theta_1 \sin^2 \Delta_{atm} \quad (9)$$

$$P(\nu_\mu \leftrightarrow \nu_\tau) = c_1^4 \sin^2 2\theta_2 \sin^2 \Delta_{atm} \quad (10)$$

Solar

$$P(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta_1 - c_1^4 \sin^2 2\theta_3 \sin^2 \Delta_{sun} \quad (11)$$

When  $\theta_1 = 0$ , the atmospheric and solar oscillations decouple and  $\nu_e$  does not oscillate in atmospheric and long-baseline experiments. Best fits [13] to the SuperK data are obtained with  $\theta_1 = 0$ ,  $\theta_2 = \pi/4$  with  $2\sigma$  bounds of  $\theta_1 < 17^\circ$  and  $|\theta_2 - 45^\circ| < 13^\circ$ . The angle  $\theta_3$  is determined by the solar data (for the just-so [10] or the MSW solutions [12], whichever is chosen by experiment). Thus we already have achieved the partial reconstruction of the neutrino MNS matrix!

## V BI-MAXIMAL MIXING MODEL

If the atmospheric and solar data are both described by maximal mixing, then there exists a unique mixing matrix for three flavors [15],

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (12)$$

The off-diagonal oscillation probabilities in this model are

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \Delta_{\text{ATM}} - \frac{1}{4} \sin^2 \Delta_{\text{SUN}} \quad (13)$$

$$P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2 \Delta_{\text{SUN}} \quad (14)$$

$$P(\nu_e \rightarrow \nu_\tau) = \frac{1}{2} \sin^2 \Delta_{\text{SUN}} \quad (15)$$

where  $\Delta \equiv 1.27\delta m^2 L/E$ . A variety of models with nearly bi-maximal mixing have also been considered [15].

## VI UNIFIED MODELS

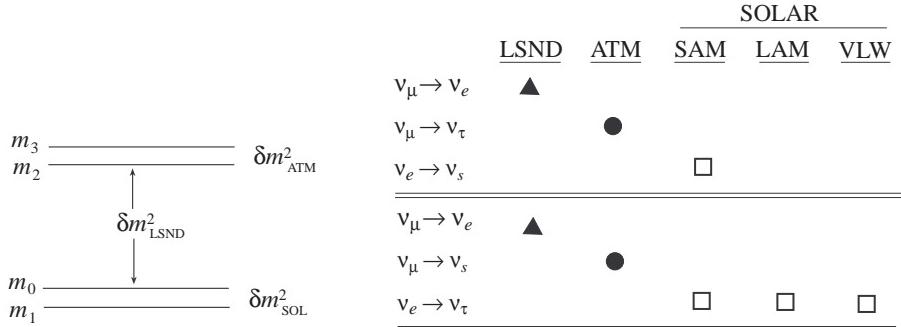
In unified models based on SO(10) [16], SU(5) [17], flipped SU(5) [18], or anomalous U(1) [19], large  $\nu_\mu \rightarrow \nu_\tau$  oscillations are accommodated but the small-angle MSW solar solution is required. This prediction is a clear distinction from the bi-maximal mixing model.

## VII DISTINGUISHING SOLAR $\nu$ -OSCILLATION SCENARIOS

The solar  $\nu$  oscillation solutions will eventually be distinguished by use of all the following measurements: (i) time-averaged total flux, (ii) day-night dependence (earth-matter effects), (iii) recoil electron energy spectra in  $\nu e \rightarrow \nu e$  events, (iv) seasonal variation, and (v) the neutral-current to charged-current event ratio (SNO experiment). The non-observation of a day-night effect has already ruled out substantial regions of the  $\delta m^2, \sin 2\theta$  parameter space [13]. The electron energy distribution from recent SuperKamiokande data (708 days) now favor the vacuum long-wavelength interpretation and nearly exclude the MSW solutions [20] unless an enhanced hep flux component is involved [21]. The day-night dependence due to the Earth is possible only for MSW solutions and has already ruled out substantial regions of otherwise allowed  $\delta m^2, \sin^2 2\theta$  values. A seasonal variation beyond the  $1/r^2$  dependence of the flux is another characteristic of only long-wavelength vacuum oscillations and the 708-day SuperK data seem to show a seasonal effect above the eccentricity correction. The neutral current measurements of the SNO experiment will distinguish active from sterile oscillations.

## VIII FOUR-NEUTRINO OPTIONS

With four neutrinos it is possible to account for the LSND data as well as the atmospheric and solar data. The preferred mass spectrum is two nearly degenerate mass pairs separated by the LSND scale [4,5]; see Fig. 2. The figure also shows the options for oscillation solutions to all data. The alternative of a 1 + 3 mass hierarchy with one heavier mass scale separated from three lighter, nearly degenerate states is disfavored when the null results of reactor and accelerator disappearance experiments are taken into account.



**FIGURE 2.** Neutrino mass spectrum showing which mass splittings are responsible for the LSND, atmospheric, and solar oscillations, and four-neutrino oscillation possibilities.

Neutrino mass matrices have been proposed [22] that can account for the observations.

## IX LONG-BASELINE EXPERIMENTS

Long-baseline experiments with  $L/E \approx 10\text{--}10^2$  km/GeV will measure  $P(\nu_\mu \leftrightarrow \nu_\tau) \simeq \sin^2 \Delta_{\text{ATM}}$  and test the atmospheric oscillation result. In addition, the existence of  $\nu_\mu \rightarrow \nu_e$  and  $\nu_e \rightarrow \nu_\tau$  oscillations may be tested. For this purpose intense neutrino beams are required. The MINOS experiment (Fermilab to Soudan) could confirm the SuperK  $\nu_\mu \rightarrow \nu_\tau$  parameter region with  $4\sigma$  sensitivity, provided that  $\delta m_{\text{ATM}}^2 > 2 \times 10^{-3}$  eV $^2$ . The K2K experiment is sensitive to  $\delta m^2 \gtrsim 3 \times 10^{-3}$  eV $^2$ .

In the future, a special purpose muon storage ring could provide high intensity neutrino beams with well-determined fluxes that could be directed towards any detector on the earth [23,24]. It could be possible to store  $\sim 10^{21}$   $\mu^+$  or  $\mu^-$  per year and obtain  $\sim 10^{20}$  neutrinos from the muon decays. Oscillations give “wrong sign” leptons from those produced by the beam. For example,  $\mu^-$  decays give  $\bar{\nu}_e$  and  $\nu_\mu$  fluxes so detection of  $\mu^+, e^-, \tau^\pm$  leptons tests for  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu (\bar{\nu}_\tau)$  and  $\nu_\mu \rightarrow \nu_e (\nu_\tau)$  oscillations. Taus can be detected via their  $\tau \rightarrow \mu$  decays and the  $\tau$ -charges so determined to distinguish  $\nu_\mu \rightarrow \nu_\tau$  and  $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$  oscillations.

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